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# Modelling of switching in surface-stabilized ferroelectric liquid crystal cells using a three-variable model in one dimension

S. M. SAID\* and S. J. ELSTON

Department of Engineering Science, University of Oxford, Parks Road,  
Oxford OX1 3PJ, UK

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The switching process in ferroelectric liquid crystal devices takes place through the formation and evolution of domains, which is modelled here using a three-variable approach. This approach includes variation of the director profile in one dimension, through the thickness of the cell. Here we discuss details of the model which are necessary in order to reproduce the domain nucleation and switching times as a function of applied voltage for both monopolar and bipolar pulses. We show that the three-variable modelling of SSFLC switching in one dimension produces excellent comparisons with experimental data for both bipolar and monopolar pulses.

## 1. Introduction

Surface-stabilized ferroelectric liquid crystal (SSFLC) structures, which were discovered by Clark and Lagerwall [1], are able to reorientate on the application of a d.c. field due to the coupling between the spontaneous polarization and the applied electric field. The switching behaviour in SSFLC cells makes them an attractive alternative to twisted nematic displays in terms of multiplexibility, increased viewing angle, lower power consumption and faster switching speeds [2]. However, there has been relatively little progress in the development of commercial ferroelectric liquid crystal display devices. One of the difficulties has been the lack of development of a full and useable continuum theory. For nematic devices, a rigorous theory has been developed, which allows both understanding of various device parameters and accurate prediction of electro-optic performance. In contrast, a full continuum theory for the behaviour of ferroelectric liquid crystals has been shown to contain more than 10 elastic and viscous constants, hence rendering a practical implementation of such a theory extremely difficult [3].

Because of this difficulty various simplified models have evolved to explain the switching processes in SSFLCs, and for use in the prediction of structures in the development of devices prior to fabrication. These models have been what could be termed low-dimensional, involving either the assumption of a uniform director profile in the device (zero dimensional in space), or allowing for variation in the profile across the thickness

of the device only (one-dimensional in space). In doing so, not only do the researchers expect to predict the behaviour of switching within SSFLC cells, but also to learn how to optimize devices to achieve particular performance goals [4]. This type of modelling has been used by Towler *et al.* [5] to investigate the effects of dielectric biaxiality in FLC switching, and by Maltese *et al.* [6] to model switching in response to various addressing waveforms. Whilst these models have achieved a considerable amount of success, they do not replicate the true physical processes that occur during the switching of surface-stabilized ferroelectric liquid crystals.

## 2. Domain switching

In real ferroelectric liquid crystal devices the switching generally takes place through the formation and evolution of domains [7]; it is therefore a multi-dimensional process. This has been experimentally investigated by a number of workers, and the process is seen to be material and device dependent. In devices the commonly used C2 structure in the smectic C layering is preferred, due to its fast electro-optic response, and the fact that switching of the cell is defined by the switching of the chevron interface, rather than the bulk of the cell.

A schematic diagram of the planar C2U configuration within an SSFLC cell is shown in figure 1. This can be formed from a liquid crystal material which shows a N–SmA–SmC phase sequence and which is confined between parallel rubbed polymer layers with a small surface pretilt. In the SSFLC cell, the liquid crystal material forms layers within which the molecular director  $\mathbf{n}$  lies on a cone of half angle  $\theta$  as shown in figure 1(a). In the SSFLC cell with a C2U geometry, a characteristic

\* Author for correspondence  
e-mail: suhana.mohd-said@eng.ox.ac.uk

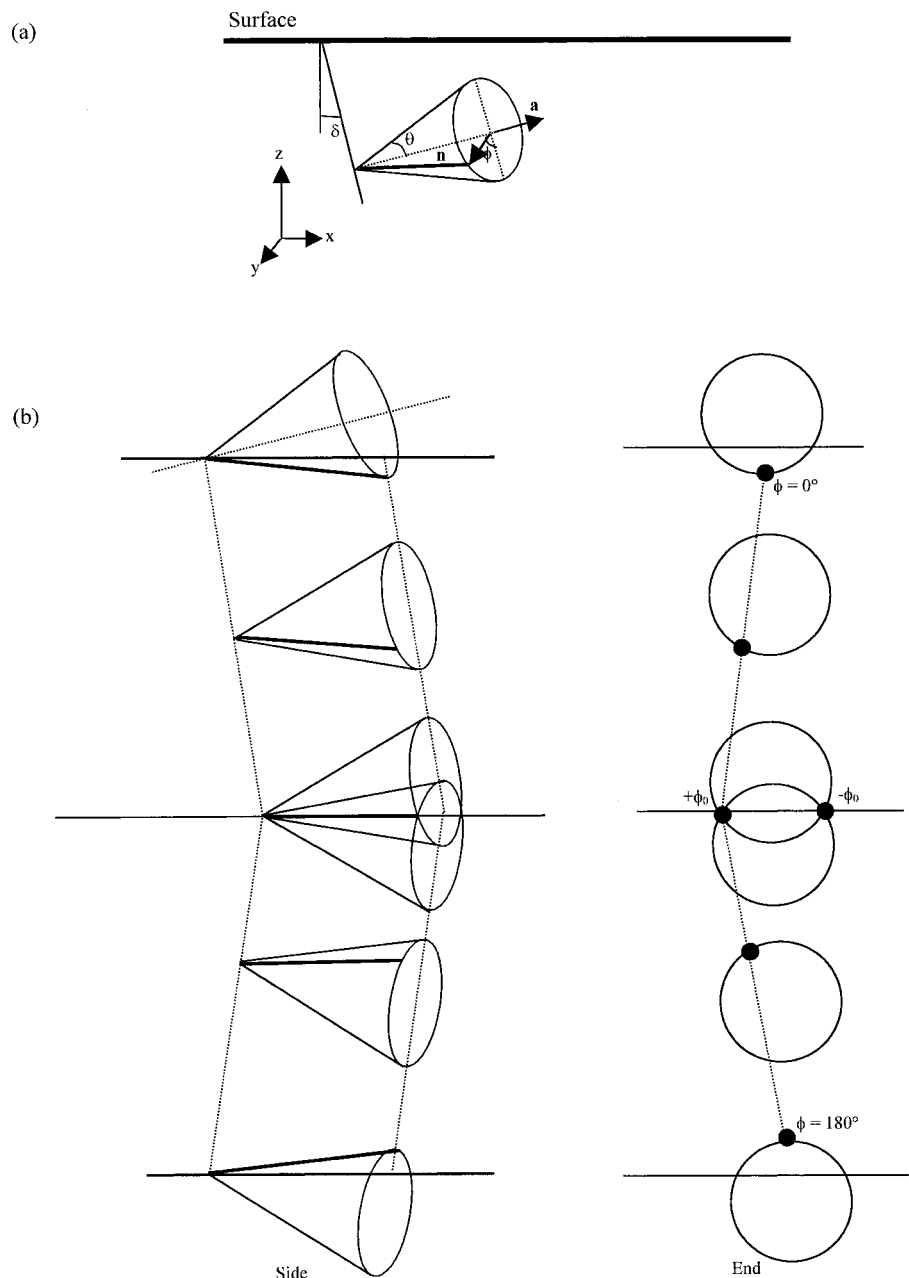


Figure 1. Schematic diagram of the planar C2U smectic C alignment geometry. (a) Illustration of the position of the molecular director  $\mathbf{n}$  on a cone of half angle  $\theta$ . (b) A characteristic chevron structure within an SSFLC cell in an ideal C2U geometry, with the side view and its corresponding end view. Layers in the top and bottom half of the cell are inclined at a tilt angle  $+/-\delta$  with a chevron interface at the centre. The end view shows the position of the director profile on the smectic cone, pinned at  $\phi = 0^\circ$  at the top surface and at  $\phi = 180^\circ$  at the bottom surface. The directors form a triangular director profile, meeting at the chevron interface at an angle  $\pm\phi_0$ .

chevron structure is formed with layers in the top and bottom half of the cell inclined at a tilt angle  $\pm\delta$  with a chevron interface at the centre [8], as shown in figure 1(b).

Switching within a C2 structure takes place as follows [9]. Before application of voltage, the device is in one of its stable states; placing a device between crossed

polarizers, this can ideally be observed as a uniform state, say white. Upon field application, the director begins to rotate around the smectic cone in response to the dipole interaction; this takes place in the bulk of the liquid crystal layer, but not in the centre of the device where the director is pinned at the chevron interface. After a certain time, the internal elastic stresses balance

the field-induced torque terms and an equilibrium is reached. Although the reorientation in this state can be considerable, if the field is removed the device will still relax back into its original state, in this case white. At about the point when equilibrium is reached domains appear. These occur when the pinned point at the chevron interface switches to the opposite state. Between crossed polarizers, black domains can be seen to appear, representing nucleation sites where the directors at the chevron interface have already switched to the opposite state. The black boat-shaped domains can be seen growing from these nucleation sites at the expense of the white state, until they finally coalesce and the whole device can be seen to have switched to the opposite black state. Removal of the field now allows the device to relax into the switched state, and the device remains black. The switching takes place through this process over a wide voltage range, although it is possible that a uniform bulk switching process takes over at sufficiently high voltages.

### 3. Switching characteristics

The domain switching process is dependent upon the polarity, the amplitude and the pulse width of the applied waveform. Hence, switching in SSFLCs may be characterized by plotting the achieved response speed as a function of the applied voltage. In particular, we are interested in the time taken to nucleate, and the time taken to switch as a function of applied voltage. Up to moderately high voltages, the electrostatic interaction between the liquid crystal molecule and the applied electric field is due to the coupling of the spontaneous polarization with the applied electric field, and a contribution due to the dielectric effects of the material. The polarization and dielectric torques are proportional to the voltage and the square of the voltage applied, respectively. For lower voltages, the polarization torque dominates: as the amplitude of the applied voltage is increased, a shorter time duration is required to switch a device completely. However, for higher voltages, the torque due to dielectric biaxiality becomes more significant, and it may take longer for the device to switch as the voltage is increased. This can be plotted as a time–voltage, or a  $t$ – $V$  curve with a minimum at  $V_{\min}$  as illustrated in figure 2 for  $t_{\text{nuc}}-V$  and  $t_{\text{sw}}-V$ .

## 4. Theoretical background

### 4.1. The three-variable concept

Three-variable modelling of the switching process in FLCs has been attempted before [10], by utilizing zero dimensional modelling of three variables,  $\phi_1$ —representing the average director orientation in the background state,  $\phi_2$ —representing the average director orientation in the switched domains, and  $A$ —the ratio of switched

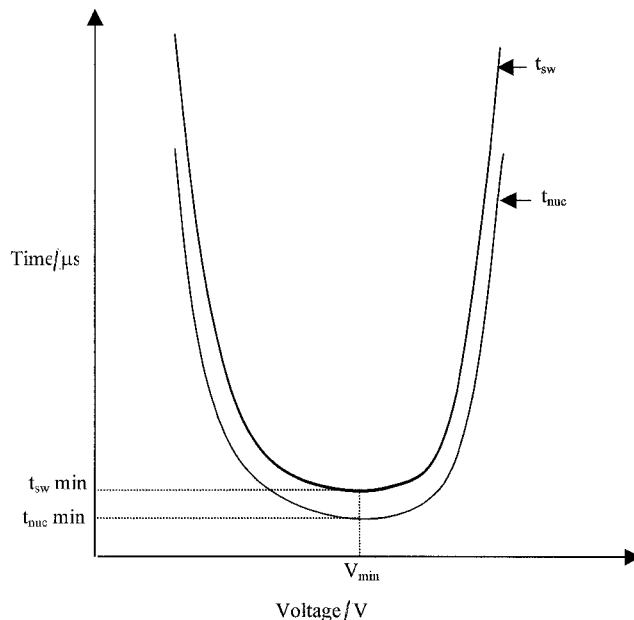


Figure 2. A typical time–voltage response for an SSFLC cell. The  $t_{\text{nuc}}-V$  curve describes the time taken to nucleate for a range of applied voltages, and the  $t_{\text{sw}}-V$  curve describes the time taken for the cell to switch fully from one uniform state to the other for a range of applied voltages.

domain area to total area of the device. This model was used to predict the monopolar and bipolar switching response in surface-stabilized ferroelectric liquid crystal devices, and achieved a certain degree of success. For example, this model successfully reproduced domain nucleation and switching times for monopolar and bipolar pulses. However, in order to reproduce experimental data correctly, the model must incorporate a complex empirical elastic stress term, because the basic elastic stress term initially proposed was insufficient to describe the elastic stress effect in a real device.

We believe that this approach can be significantly improved, and therefore accurately predict switching characteristics in ferroelectric liquid crystal cells, by extending the model into one dimension, through the thickness of the cell, whilst retaining the calculation of the ratio of the switched domain area. It is expected that this model will then accurately predict the behaviour of the SSFLC cell during switching as it simulates the actual reorientation of the director profile under application of the electric field, whilst retaining a low number of governing parameters. The relative simplicity of this model would also imply a short computational time when calculating the reorientation and transmission behaviour of the cell.

### 4.2. Modelling in one dimension

In order to describe realistically the domain switching process, which is initiated by latching of the chevron

interface from one equilibrium position to the other, we need to model in one dimension through the thickness of the cell. For a cell that appears uniform in optical texture, we assume no variation of the director profile in the directions parallel to the surface of the cell.

Assuming uniformly tilted, incompressible layers the director reorientation  $\phi(z)$ , (for the domain or background) will be modelled in each half of the chevron structure using a simple equation of the form:

$$\eta_b \frac{\partial \phi}{\partial t} = K \frac{\partial^2 \phi}{\partial z^2} + \mathbf{P}_s \mathbf{E} \cos \phi \cos \delta - \partial \varepsilon \varepsilon_0 \mathbf{E}^2 \cos \phi \sin \phi \cos^2 \delta \quad (1)$$

where  $\eta_b$  is the rotational viscosity within the bulk of the SSFLC cell,  $\phi(z)$  is the azimuthal angle (background or domain),  $\mathbf{P}_s$  is the spontaneous polarization of the material,  $\mathbf{E}$  is the applied electric field,  $\delta$  is the chevron tilt angle,  $K$  is the elastic constant of the material and  $\partial \varepsilon$  is the dielectric biaxiality of the material.

The director within the device is assumed to be fixed at the liquid crystal–glass boundary, with the angle  $\phi = 0^\circ$  at the top surface, and  $\phi = 180^\circ$  at the bottom surface. The intersection of the two smectic cones at the chevron interface is equivalent to the intersection of a horizontal plane with either cone. Hence, the chevron interface can be treated as an internal surface with minimal energy when the director lies in the plane parallel to the sides of the cell. The chevron interface may therefore be described by a surface torque:

$$T = T_0(2 \cos \phi_0 \sin \phi - \sin 2\phi) \quad (2)$$

where  $T_0$  is a chevron interface torque.

The intrinsic bistability of equation (2) leads to two stabilized equilibrium positions, which are the two stable azimuthal director angles at the chevron interface,  $+\phi_0$  and  $-\phi_0$ , as shown in figure 1. Latching is then determined to occur when the director at the chevron interface reaches the opposite stable state.

### 5. Three-variable modelling in one dimension

In order to reconcile the chevron interface latching using a one dimensional model, with the domain growth process, we incorporate the one dimensional model within the three variable concept.  $\phi_1(z)$  represents the director orientation for the background state and is a function of the distance across the cell, which we choose to be in the  $z$  direction. The director profile throughout the thickness of the cell in this model is assumed to begin in a Triangular Director Profile in its relaxed state.  $\phi_2(z)$  represents the director orientation within the switched domains through the thickness of the cell.  $A$  again represents the ratio of the domain area to total device area.

#### 5.1. Latching of the chevron interface

Initially, the SSFLC cell is in a relaxed stable state, i.e. there is no difference between the background and domain states, and hence their initial director profiles are identical. As an electric field is applied, the directors start to reorient, according to the governing equation (1). At certain points, the chevron interface torque is overcome, and the director at the chevron interface switches from  $+\phi_0$  to  $-\phi_0$ , as described by equation (2). This leads to the nucleation of domains at these points. With reference to the  $t$ – $V$  curves in figure 2, the time taken for these nucleation sites to appear is defined as  $t_{\text{nuc}}$ . In order to distinguish between the background and domain regions, we propose that the nucleation sites possess a weaker chevron interface torque compared with the background regions, and are therefore easier to switch from one equilibrium position to the other. Hence, the torque-balance equations at the chevron interface for the background and domain regions are defined as follows:

$$\eta_b \frac{\partial \phi}{\partial t} = K \frac{\partial \phi}{\partial z} + T_0(2 \cos \phi_0 \sin \phi - \sin 2\phi) \quad \text{for the background region} \quad (3)$$

$$\eta_s \frac{\partial \phi}{\partial t} = K \frac{\partial \phi}{\partial z} + T_{\text{ch}}(2 \cos \phi_0 \sin \phi - \sin 2\phi) \quad \text{for the domain region} \quad (4)$$

where  $\eta_s = \eta_b \times dz$ ,  $dz$  being the thickness of the chevron interface layer [11], and  $T_{\text{ch}} < T_0$ , where we assume that there is a narrow distribution of threshold torques centred around  $T_{\text{ch}}$  at the nucleation sites, and therefore a fixed  $T_{\text{ch}}$  can be used. This is reasonable, as in practice all domains are seen to nucleate within a narrow time slot.

#### 5.2. Domain growth

Upon latching at the chevron interface, the domains start to grow from the nucleation sites. Therefore, in order to represent the switching completely, we also require a model for domain area evolution. A simple solution would be to suggest that the area variation is linear with time, with a proportionality factor depending on the applied voltage. This factor would then incorporate physically observed phenomena in the respective voltage regimes, for example the number of nucleated domains and variation in domain growth rate. However, it also needs to be taken into consideration that the domains coalesce during the evolution. Here we take the example from the polarization reversal process in solid ferroelectric materials, by which domains form and grow upon a change in the polarization state of the material. We therefore follow the suggestion of Ishibashi [12]

and use the Avrami model, which is a statistical model which takes into account the number of nucleation sites and domain wall growth. This formulation assumes that the domain growth process is two dimensional and isotropic. The Avrami model of domain area can be written as:

$$A = 1 - \exp \left\{ -N \left[ \frac{\sigma}{2} v (t - t_{\text{nuc}})^2 \right] \right\} \quad (5)$$

where  $A$  is the fraction of switched area,  $N$  is the number of domains which nucleated at time  $t_{\text{nuc}}$ ,  $\sigma$  is a shape factor and  $v$  is the domain wall velocity.

Equation (5) can be differentiated to obtain a form for the instantaneous rate of change of area:

$$\frac{dA}{dt} = Rv \sqrt{N} (1 - A) \left[ \ln \left( \frac{1}{1 - A} \right) \right]^{1/2} \quad (6)$$

where  $R$  is a proportionality constant.

Previous workers have assumed that the parameters  $N$ , the number of nucleation sites produced, and  $v$ , the domain wall velocity, are both directly proportional to the applied voltage [4, 9]. However, for a material of high biaxiality such as SCE8, we anticipate that the effects of both the polarization torque and dielectric biaxiality torque would influence the behaviour of these two parameters. Therefore, we have set both  $N$  and  $v$  to be dependent upon the net torques acting on the device, namely the resultant torque due to polarization coupling and dielectric biaxiality. The net torque acting on the device is expressed as:

$$\text{Net torque} = \mathbf{P}_s \mathbf{E} \cos \phi \cos \delta - \partial \varepsilon \varepsilon_0 \mathbf{E}^2 \cos \phi \sin \phi \cos^2 \delta.$$

$R$  then absorbs the proportionality factor between the domain wall velocity ( $v$ ) and the number of nucleated domains ( $N$ ) and takes into account the shape factor.

In order to implement this equation, it is assumed that at nucleation a small, finite area exists, and that at a larger area switching occurs. An arbitrary starting area of 0.0001 and a finishing area of 0.9999 are chosen to avoid  $da/dt \rightarrow 0$  at any time. The time taken for the domain area to grow completely to the opposite state; i.e. to switch fully, is then defined as  $t_{\text{sw}}$ , as indicated in figure 2.

## 6. Numerical implementation

The governing equations above were translated into a discrete form, to allow for numerical calculation of the switching process. The device was modelled by dividing the FLC into a finite number of thin layers across the thickness of the cell, and computing the value of  $\phi$  within each layer. The value of  $\phi$  at each position was calculated using the forward difference method in the time domain and the central difference method in the space domain.

Figure 3 illustrates the progression of  $\phi_2$  and  $A$  with respect to time, to demonstrate the mechanism of the three-variable model;  $\phi_1$  and  $\phi_2$  represent the actual azimuthal angle at the chevron interface for the background and domain regions, respectively. Initially, both  $\phi_1$  and  $\phi_2$  start in the same position at the chevron interface. As an electric field is applied,  $\phi_1$  remains at its initial stable position at the chevron interface, whilst  $\phi_2$  tends towards the opposite state and finally latches onto the opposite stable state (i.e.  $\phi_2 = +\phi_0$ ) at  $t = t_{\text{nuc}}$ , the time to nucleation. It can be noted that once the director has latched to the opposite state, the area growth commences and is completed at  $t = t_{\text{sw}}$ , the time taken for the cell to switch fully to the opposite state.

The resultant director profile reorientation throughout the thickness of the cell upon the application of an electric field is illustrated in figures 4(a) and 4(b) for the background and domain regions, respectively. In figure 4(a) it can be observed that although the bulk reorientation tends towards the opposite state, the chevron interface remains pinned to its original position and hinders switching. Figure 4(b), on the other hand, illustrates that both the bulk and chevron interface switch to the opposite state, due to the weaker chevron interface torque in the domain region.

## 7. Transmission

Once the director profile reorientation within the SSFLC cell has been calculated, the transmission through the domains and the background can be obtained from the director profile for the background and domains by feeding each into a separate Berreman  $4 \times 4$  matrix

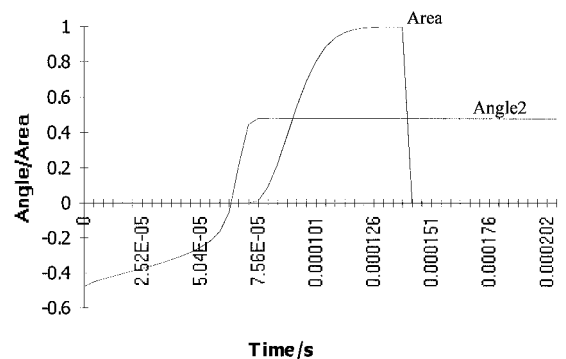


Figure 3. Director reorientation in time, for a layer of directors adjacent to the chevron interface. Angle 2 represents the domain director profile which latches onto the opposite stable state, hence initiating domain area growth at  $t = t_{\text{nuc}}$ . It can be noted that once the director has latched to the opposite state the area growth commences, also at  $t = t_{\text{nuc}}$ .

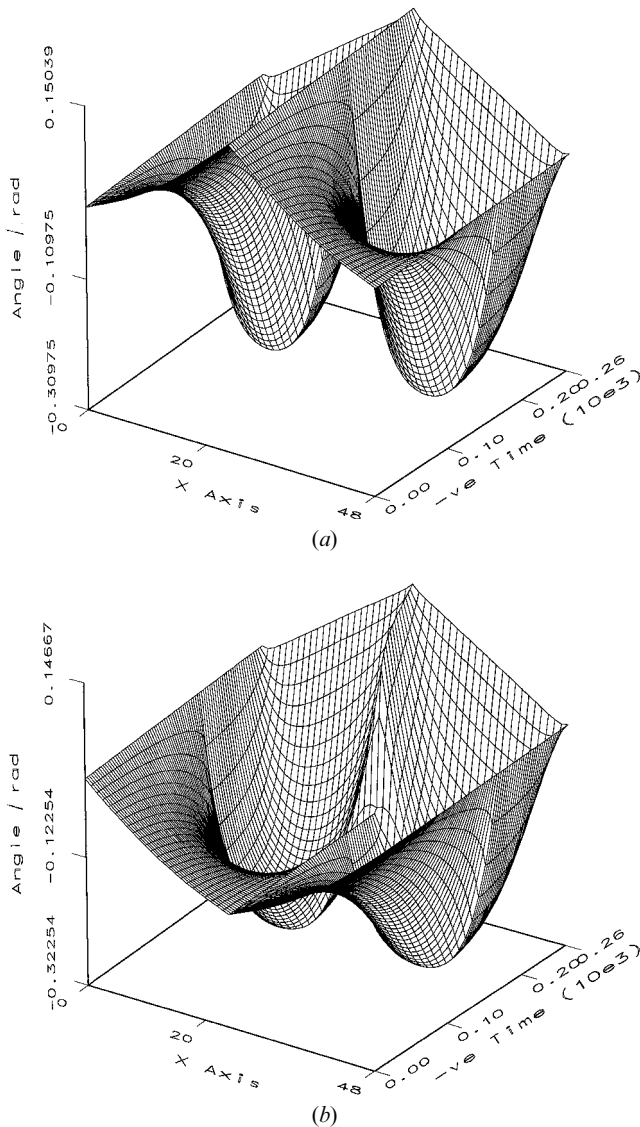


Figure 4. Director profile reorientation for the background and domain regions through the thickness of the cell. Initially, both the background and domain are in a similar Triangular Director Profile configuration, at  $t = 0$ . (a) As a monopolar pulse is applied, the director at the chevron interface for the background remains pinned, whilst most of the bulk reorients to the opposite state. (b) As a monopolar pulse is applied, the director at the chevron interface first remains pinned to its initial state, then latches to the opposite state at  $t = t_{\text{uc}}$ . When the monopolar pulse is removed (i.e.  $V = 0$ ), the director profile relaxes into its newly switched state.

program. The resultant transmissions for both the background and domain are then multiplied by its area and added together to represent the overall transmission through the device. The calculation process to obtain transmission is illustrated in figure 5.

The three-variable model in one dimension is able to reproduce the different regimes of switching, as follows:

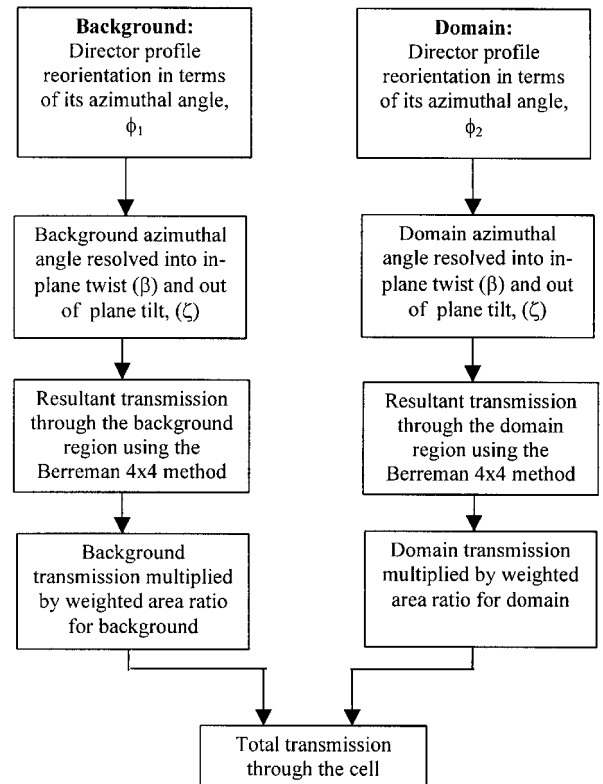


Figure 5. Flowchart describing the calculation of transmission through the SSFLC cell based on the three-variable model in one dimension.

- (i) The non-switching state; caused by removal of voltage before the chevron interface has had a chance to latch to the opposite stable state. Elastic relaxation upon removal of voltage causes the director to relax back to its initial stable state.
- (ii) The fully switched state; caused by a voltage of sufficient amplitude (under a fixed pulse width) causing switching of the chevron interface to the opposite stable state, and full switching of the cell via domain growth.
- (iii) The partially switched state; caused by a voltage of sufficient amplitude to cause switching of the chevron interface, and initiate domain growth, but this voltage is removed before domain growth to the opposite state is completed. This can be observed by polarizing optical microscopy as a mixture of light and dark states, which remain for a time period of some seconds even after the voltage has been removed. In our model, this is equivalent to maintaining the area at its partially switched ratio, even after the voltage has been removed.

## 8. Experiment

The experimental set-up was as follows: A  $1.5\mu\text{m}$  thick device filled with SCE8 in a C2 stabilized structure

was placed on the observation stage of a polarizing microscope, with polarizers set perpendicular to each other. The cell was oriented so that the alignment direction was at  $22.5^\circ$  to the polarizer angle, and kept at an ambient room temperature of  $23.0^\circ$ . The device was illuminated with a non-coherent light source. The experiment was in two parts. The first part was concerned with characterizing the time to nucleate,  $t_{\text{nuc}}$ , and the time to switch fully,  $t_{\text{sw}}$ , as a function of applied voltage. The cell was observed using the polarizing microscope, and the resulting image was captured by a charge-coupled device camera connected to a video monitor. The camera was focused on a defect-free region of the device.

A waveform generated by an arbitrary waveform generator, representing either a monopolar or bipolar pulse was applied to the device. A d.c. balanced, cyclic waveform was designed to avoid ionic build up in the SSFLC device. For a given voltage, the pulse width of the waveform was varied to determine the threshold time,  $t_{\text{nuc}}$ , when the domains just started to nucleate, and  $t_{\text{sw}}$ , which is the time taken for the device to switch completely to the opposite state. The former can be defined by the threshold pulse width needed to cause the region of the device under observation to start flickering—this corresponds to the fact that the domains have started to nucleate and partial switching is starting to take place. On the other hand,  $t_{\text{sw}}$  is defined as the pulse width needed to cause the device to stop flickering: this is an indication that full switching has been achieved.

The second part of the experiment studied the transmission response of the switched cell between crossed polarizers. Particular note was taken of the characteristics of the response, in particular its shape, and voltages which would cause switching, non-switching and partial switching conditions.

## 9. Experimental results

The experimental data for  $t_{\text{nuc}}$  and  $t_{\text{sw}}$  for the monopolar and bipolar cases are shown in figure 6. The bipolar plots are of particular interest, as they produce two sets of  $t_{\text{nuc}}$  and  $t_{\text{sw}}$  for each voltage range. Under the polarizing microscope, this is observed as the appearance of flickering domains at a particular frequency ( $f_{\text{nuc}1} = 1/t_{\text{nuc}1}$ ), and the disappearance of the flickering domains at a lower frequency when full switching has been achieved ( $f_{\text{sw}1} = 1/t_{\text{sw}1}$ ). However, when the frequency is reduced even further, partial switching is seen again ( $f_{\text{nuc}2} = 1/t_{\text{nuc}2}$ ), followed by full switching at an even lower frequency ( $f_{\text{sw}2} = 1/t_{\text{sw}2}$ ). The double appearance of switching within one voltage range can be explained by the two possible switching mechanisms in a bipolar pulse scheme. The first possibility is that latching

at the chevron interface is caused by the first pulse of the bipolar scheme, and the second pulse simply perturbs the new state into which the device has switched. This corresponds to switching the director from a relaxed state, and can be referred to as leading pulse switching [13]. The second possibility is that the first pulse simply stresses the director profile, but it is the second pulse that causes latching at the chevron interface to the opposite state. This corresponds to switching the director from a stressed state, and can be referred to as trailing pulse switching [13]. It is to be expected that the trailing pulse switching (from a stressed state) would require a longer pulse duration to switch the chevron interface, and therefore this would correspond to the switching range at lower frequency ( $t_{\text{nuc}2}$  and  $t_{\text{sw}2}$ ). By similar argument, we would expect the leading pulse switching to correspond to the range  $t_{\text{nuc}1}$  and  $t_{\text{sw}1}$ .

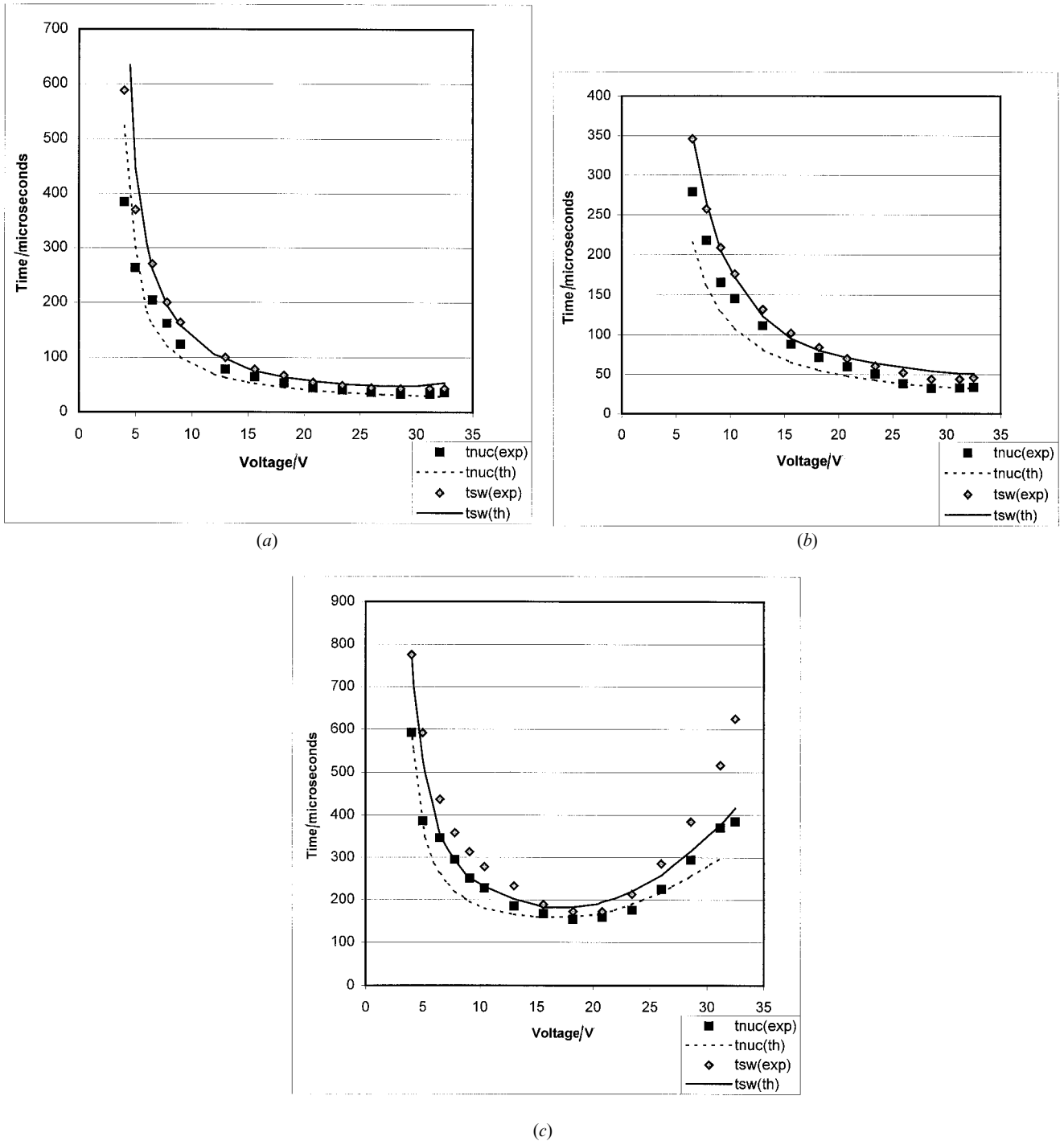
## 10. Choice of modelling parameters

In order to reproduce the switching process using three-variable modelling in one dimension, we utilize a number of parameters obtained by other workers [14, 15]. These include the spontaneous polarization ( $\mathbf{P}_s$ ), smectic cone angle ( $\theta$ ), layer tilt angle ( $\delta$ ), device thickness ( $d$ ), and elastic constant ( $K$ ). Some other parameters, such as  $T_0$  and  $T_{\text{ch}}$ , are peculiar to this model and together with  $\partial\varepsilon$  and  $\eta_b$  are treated as ‘free parameters’ in order to obtain reasonable correlation between experimental data and theory.

Firstly, a good theoretical fit to the experimental  $t_{\text{nuc}}-V$  curve for both the monopolar and bipolar cases is necessary, as this provides a ‘baseline’ for the calculations of  $t_{\text{sw}}$ . An outline of the steps taken to ensure a good fit between the data and theory is now given:

- (1) Determine the basic material and device dependent parameters.
- (2) Adjust  $\partial\varepsilon$  to ensure that  $V_{\text{min}}$  of the theory corresponds to  $V_{\text{min}}$  for the experimental data, and subsequently ensure that the high voltage response for the theory fits the experimental data.
- (3) Adjust  $T_{\text{ch}}$  to ensure that the switching of the director at the chevron interface within the domains occurs at suitable  $t_{\text{nuc}}$  for low and high voltage regimes.
- (4) Adjust  $T_0$  so that the director at the chevron interface remains latched at the initial stable states so as to hinder switching of the background region in the corresponding voltage regimes.
- (5) Adjust the bulk viscosity coefficient,  $\eta$ , to ensure that the time range for the theoretical  $t_{\text{nuc}}-V$  plots corresponds with the time range for the experimental data.





(a)

(b)

(c)

Figure 6. Theoretical fitting to experimental data: (a) monopolar pulse data; (b) leading edge bipolar pulse data; (c) trailing edge bipolar pulse data.

Once a fit of theory to experiment for  $t_{nuc}-V$ , for both monopolar and bipolar voltages, has been obtained, the area growth rate is adjusted to obtain the correct  $t_{sw}-V$ . A complete list of the parameters used in our three variable model in one dimension is shown in the table.

### 11. Comparison of results for experiment and theory

For both the monopolar and bipolar cases, the theoretical predictions for  $t_{nuc}-V$  and  $t_{sw}-V$  are represented by the continuous lines in figure 6, and produce a reasonable fit to the experimental curves. However, upon close

Table. List of Parameters used in the Three Variable Model in One Dimension.

Parameter	Value
$P_s$	$6.3 \text{ nC cm}^{-2}$
$K$	$1.0 \times 10^{-11} \text{ N}$
$\theta$	$19.02^\circ$
$\delta$	$17.02^\circ$
$\partial\epsilon$	$0.35$
$\Delta\epsilon$	$-1.91 \times 10^{-11}$
$\eta$	$0.055 \text{ N sm}^{-2}$
$T_o$	$0.055 \text{ N m}^{-1}$
$T_{ch}$	$1.485 \times 10^{-3} \text{ N m}^{-1}$
$d$	$1.5 \mu\text{m}$

scrutiny, it may be noted that in figure 6 the theoretical curves are somewhat ‘broader’ than the experimental curves, i.e. the theoretical predictions may be compatible with the experimental results at low voltages, but for medium to high voltages, the director profile is switching too quickly in the theoretical predictions. This difference between the experiments and the theory may be due to an oversimplification of our existing model. For example:

- (1) A one-elastic constant approximation may be inadequate to model the respective bend, splay and twist components of the elastic forces prevalent during the switching of the SSFLC cell.
- (2) A uniform electric field approximation may be inadequate to model the SSFLC cell which has a non-uniform director profile.
- (3) The effect of dielectric anisotropy ( $\Delta\epsilon$ ) has not been considered, even though according to dielectric permittivity measurements for SCE8 [15], the magnitude of the dielectric anisotropy is significant (at  $T = 20^\circ\text{C}$ ,  $f = 1.0 \text{ kHz}$ ,  $\Delta\epsilon = -1.65$ ).

The importance of each of these effects in a one-dimensional model was investigated, and it was found that the dielectric anisotropy effect would be the most significant influence that is ignored in the above model. The material SCE8 has a negative anisotropy, which couples to the applied electric field, causing a torque on the director around the cone tending to reduce the out-of-plane tilt, and hence preserve the planar configuration within the cell. Hence, the inclusion of a dielectric anisotropy term would produce a more accurate description of the director profile throughout the thickness of the SSFLC cell. Therefore, we modify the existing governing equation to include the effect of dielectric anisotropy, and test the model to see if it produces a better fit to the experimental results. The new governing equation

can be written as:

$$\eta \frac{\partial\phi}{\partial t} = K \frac{\partial^2\phi}{\partial t^2} + \mathbf{PE} \cos\phi \cos\delta - \partial\epsilon\epsilon_0 \mathbf{E}^2 \cos\phi \sin\phi \cos^2\delta - \epsilon_0 \mathbf{E}^2 \Delta\epsilon \cos\phi \times \left[ \frac{1}{4} \sin 2\theta \sin 2\delta - \sin\phi \cos^2\delta \sin^2\theta \right]. \quad (7)$$

## 12. A modified model

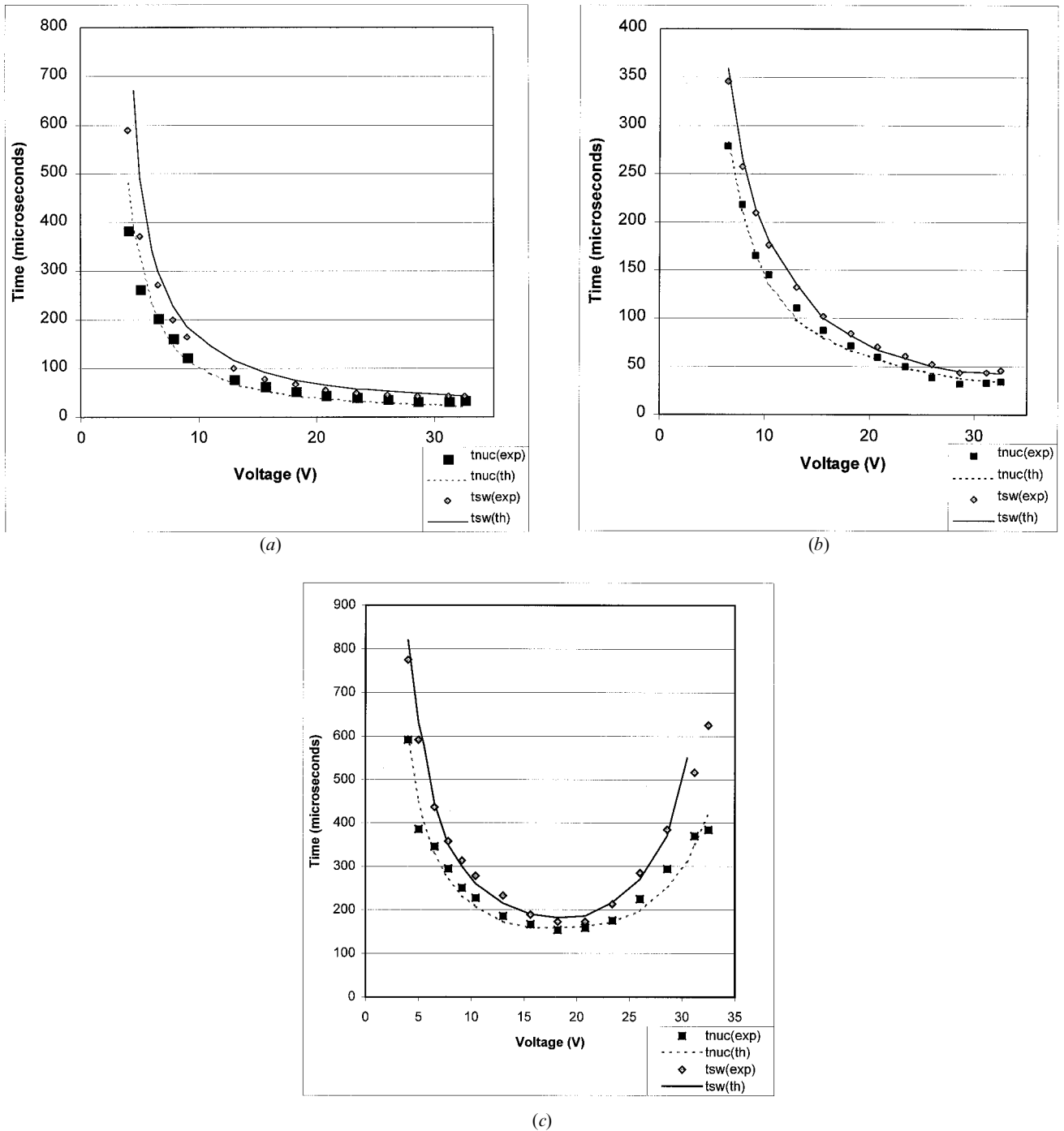
Equation (7) was translated into a discrete form and numerically solved to produce the director profile reorientation under the application of an applied field. Parameter values from the previous model were retained and the value for the dielectric anisotropy, which was negative, was obtained from detailed measurements by Brown and Jones [15]. In keeping with the procedure of the original model, equations (3) and (4) were used to simulate latching at the chevron interface.

It was observed that, for all ranges of voltages for the monopolar pulse, an excellent fit of theory to experiment for the  $t_{nuc}-V$  curves was obtained using the three-variable model in one dimension incorporating the effects of spontaneous polarization coupling, dielectric biaxiality and dielectric anisotropy. Similarly, the three-variable model in one dimension produced a successful fit to the  $t_{nuc}-V$  experimental data, for both the leading edge bipolar and trailing edge bipolar pulses.

Using the successful  $t_{nuc}-V$  plot as a baseline, the domain area progression was recalculated. Both parameters  $N$  and  $v$  for domain area growth were then made to be dependent upon the net torques acting on the device, which are now the resultant torque due to the competing effects of the polarization torque, the dielectric biaxiality torque and the dielectric anisotropy torque. The resultant  $t_{sw}-V$  theoretical plots also provide excellent fits to both monopolar and bipolar pulses, for all voltage ranges. This confirms the success of the three-variable model in one dimension in predicting the response of the SSFLC cell under the influence of simple electrical pulses, as shown in figure 7. It should be noted that the fitting of theory to experimental data has been obtained from experiments carried out at a fixed temperature. Variation of temperature would entail a more complex analysis of the respective parameters used in this model and how they vary with temperature. This is however within the capabilities of the approach used.

## 13. Transmission at high voltages

The transmission responses for both the monopolar and bipolar pulses were simulated by the three-variable model in one dimension, for a range of voltages. To illustrate this we show some interesting transmission characteristics at high voltages. With reference to figure 8,



(c)

Figure 7. Theoretical fitting to experimental data, using a modified Three-Variable Model in One Dimension incorporating the effects of dielectric biaxiality and dielectric anisotropy: (a) monopolar pulse data; (b) leading edge bipolar pulse data; (c) trailing edge bipolar pulse data.

the theoretical transmission plot for a highly biaxial material with an asymmetric bipolar pulse of amplitude 33.5 V demonstrates that the cell did not switch, due to the hindrance of switching at the chevron interface because of the high biaxiality. However, interestingly, even though no switching occurred, the transmission plot

showed a decrease in intensity when the voltage was reversed; this seemed to rise again when the voltage was removed. We can explain the observation as follows: upon forward stressing, it can be observed that the regions near the chevron interface are hindered from switching to the opposite side. However, the director

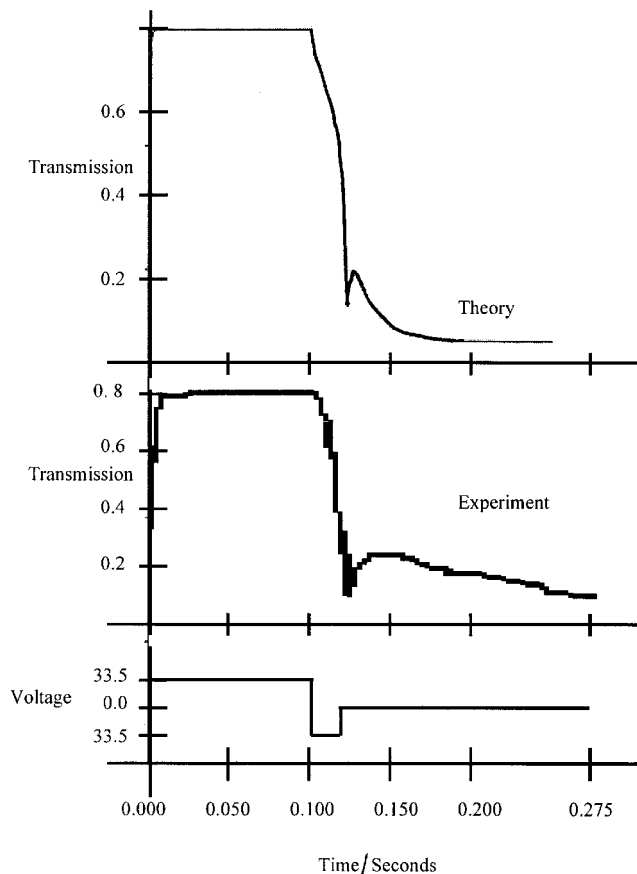


Figure 8. Comparison of theory and experiment for the transmission response to a high voltage bipolar pulse.

profile near the surface (which experiences the maximum coupling of spontaneous polarization to the applied electric field) is able to reorient towards the opposite stable state. This corresponds to the decrease in intensity upon reverse stressing. When the voltage is totally removed, the director profile (which has not succeeded in switching to the opposite side) relaxes to its original position, and hence correspondingly shows a slight rise in transmission before relaxing into the original triangular director profile. This particular effect upon transmission for a high voltage bipolar pulse is correspondingly reproduced in experiment, as also shown in figure 8. This result also shows the power of the three-variable model

in one dimension, which is able to reproduce unique transmission effects due to the effect of the director profile reorientation.

#### 14. Conclusions

In this paper we have shown that the Three-Variable Theory in One Dimension has proved to be a relatively simple but accurate tool for predicting the switching behaviour of an SSFLC cell. It has successfully modelled the switching response for both monopolar and bipolar switching for a material with a high biaxiality, for a wide range of voltages. Since this model incorporates the dielectric biaxiality and anisotropy torques in its governing equations and has proved to be successful in duplicating experimental results in the region of the  $t$ - $V$  minimum, it should be possible, by its use, to simulate the response of high voltage addressing schemes.

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